

## EFFECT OF ROUND ORIFICE ASPECT RATIO ON NON-NEWTONIAN FLUID DISCHARGE FROM TANKS

Tsepang Mohajane<sup>1</sup>, Morakane Khahledi<sup>1</sup>, Rainer Haldenwang<sup>1</sup>,  
Veruscha Fester<sup>1</sup>, Raj Chhabra<sup>1,2</sup>

<sup>1</sup>Dept. Civil Engineering and Surveying, Cape Peninsula University of Technology, Cape Town, South Africa; tsepangmohajane@gmail.com; <sup>2</sup>Department of Chemical Engineering, Indian Institute of Technology Ropar, India

Non-Newtonian fluids have complex viscous characteristics, which make it difficult to measure the flow rate during their processing and handling. Several researchers have conducted studies on the flow rate measurement of non-Newtonian fluids using orifices installed in horizontal pipes. However, the same cannot be said for the gravitational flow of non-Newtonian fluids through circular orifices from the bottom of tanks. The aim of this work was to determine the coefficient of discharge ( $C_d$ ) values for different non-Newtonian fluids from a tank through circular orifices of varying aspect ratios (the relationship between the length and the diameter of an orifice denoted by  $L/d$ ) fitted at the bottom of the tank, as a function of fluid properties. The Flow Process Research Centre at Cape Peninsula University of Technology built a rig that was used for the experiments. Two circular orifices of aspect ratios 0 and 5 were fitted one at a time in the center of the bottom of the tank and flush with the inside surface. Water was used for calibration and various concentrations of glycerine solutions, carboxymethyl cellulose (CMC) solutions, and bentonite and kaolin suspensions were used as test fluids. The rheology of the test fluids was established using a rotary viscometer. The analysis is presented in the form of the coefficient of discharge ( $C_d$ ) against the orifice Reynolds number ( $Re$ ). While the existing literature showed that in turbulent flow, Newtonian and non-Newtonian fluids have an average  $C_d$  value of 0.62 and 0.67 respectively, the turbulent flow region of this study, shows that  $C_d$  values of both Newtonian and non-Newtonian fluids varied with orifice thickness. The flow trends of  $L/d$  of 0 are separate from the flow trends of  $L/d$  of 5 in the laminar and turbulent regions.

**KEYWORDS:** Coefficient of discharge, Non-Newtonian, Newtonian, Aspect ratio, Gravitational flow, Circular orifice

### NOTATION

$a_2$	Area of an orifice ( $m^2$ )
$C_d$	Coefficient of discharge (-)
$d$	The diameter of an orifice (m)
$Re$	Reynolds number (unitless)
$Re_{MR}$	Metzner and Reed Reynolds number (-)
$Re_2$	Slatter and Lazarus (1993) Reynolds number (-)
$K$	Fluid consistency index ( $Pa \cdot s^n$ )
$n$	Flow behaviour index (-)
$V_2$	Fluid velocity in the orifice (m/s)

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$V_1$	Fluid velocity in the tank (m/s)
$h_1$	Fluid height in the tank measured from the orifice (m)
$h_2$	Reference height at the orifice (m)
$L$	Orifice length (m)
$P_{\text{Atm}}$	Atmospheric pressure (N/m <sup>2</sup> )
$P_1$	Pressure in tank (N/m <sup>2</sup> )
$P_2$	Pressure in orifice (N/m <sup>2</sup> )
$Q_1$	The fluid flow rate in a tank (m <sup>3</sup> /s)
$Q_2$	The fluid flow rate in the orifice (m <sup>3</sup> /s)
$g$	Acceleration due to gravity (m/s <sup>2</sup> )
$\rho$	Fluid density (kg/m <sup>3</sup> )

## 1. INTRODUCTION

Non-Newtonian fluids are often encountered in food processing and chemical industries. For the appropriate flow measurement of these fluids, their rheological parameters and the coefficient of discharge of the orifice have to be known. The discharge of non-Newtonian liquids from tanks has been a subject of a few publications (Dziubiński and Marcinkowski, 2006). They used several orifice diameters with changing lengths. Their experiments were conducted from a cylindrical glass tank of 0.2 m diameter. The orifice diameters were 5, 8, 12.5 and 17 mm while  $L/d$  ratios were 0, 0.35, 0.5, 0.75, 1 and 3. Various concentrations of CMC, as a Power-Law fluid represented non-Newtonian fluids with  $Re$  ranging from 0.01 to 1000. Water, ethylene glycol and starch syrup were used as Newtonian fluids with  $Re$  ranging from 0.001 to 10000. In the laminar flow region, the  $C_d$  values increased as  $Re$  increased; however, they found that Newtonian fluids formed separate flow trends. In the turbulent flow region, the Newtonian fluids became constant at 0.62 and that of non-Newtonian fluids at 0.67 for all the aspect ratios. Yildirim (2010) stated that the  $C_d$  value of circular sharp crested orifices in a fully turbulent region is 0.61, but in reality, it varies significantly with respect to the head of water and orifice geometry.

Çobanoğlu (2008) conducted flow rate measurements experiments using water in order to establish the effect of orifice  $L/d$  ratios on the  $C_d$  values in relation to the  $Re$ . The orifice diameters were 6 and 10.35 mm and  $L/d$  ratios were 8 and 5. A rectangular tank with clear Perspex walls (0.37 m, 0.47 m base and 0.4 m height) with a hole machined at the side of a the tank was used to carry out all experiments. The  $Re$  ranged from 2300 to 20000, the head of the fluid was kept constant when conducting the experiments. The results showed that the  $C_d$  values of  $L/d$  of 8 were higher than those of  $L/d$  of 5. He found that for  $L/d$  of 5, the  $C_d$  values ranged from 0.75 to 0.83 with an average  $C_d$  of 0.79 and for  $L/d$  of 8, the  $C_d$  values ranged from 0.77 to 0.85 with an average  $C_d$  of 0.83.

Kiljanski (1993) conducted gravitational flow measurements of Newtonian fluids (glycerine, syrup and glycol) with very low viscosities from the side of the tank. The  $Re$  of the fluids ranged from 0.01 to 500. He observed the effect of orifice  $L/d$  ratios on the  $C_d$  values in relation to  $Re$ . The tests were carried out using a vertical Perspex cylindrical tank of 0.038 m diameter. He used 5 circular orifices of diameters 2, 3 and 5 mm and  $L/d$  ratios were 0, 0.5 and 1. The head of the fluid was kept constant when conducting the experiments. He found that in the range of small Reynolds numbers ( $Re \leq 10$ ) the

experimental points for each L/d ratio lie on separate straight lines with slopes of 0.5. He also found that the coefficient of discharge of Newtonian fluids is proportional to the square root of the Reynolds number.

According to Jithish and Kumar (2015) and Yildirim (2010) an orifice meter work on the principle of Bernoulli's theorem. For water flowing gravitationally out of a tank open to the atmosphere through an orifice that discharges freely into the atmosphere, Bernoulli's equation is given as:

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \quad (1)$$

where  $P_1 = P_2 = P_{Atm}$  and  $v_1 \ll v_2$

$h_1$  = the height of the liquid in the tank measured from the orifice and  $h_2$  is considered as zero because it is the reference height at the orifice.

After iteration, Equation 1 simplifies to:

$$V_2 = \sqrt{2gh_1} \quad (2)$$

Continuity equation states that  $Q_1 = Q_2$ . Therefore, the flow through the orifice is:

$$Q_2 = C_d \times a_2 \sqrt{2gh_1} \quad (3)$$

To obtain the coefficient of discharge,  $C_d$  in Eq.3 is made the subject of the formula resulting in:

$$C_d = \frac{Q_2}{a_2 \sqrt{2gh_1}} \quad (4)$$

The Newtonian fluids Reynolds number as presented in Equation 5 was used for classifying Newtonian fluid flow.

$$Re = \frac{\rho V d}{\mu} \quad (5)$$

For the flow of Power-Law model fluids, the Metzner and Reed (1955) Reynolds number was applied. Metzner and Reed (1955) Reynolds number is given by:

$$Re_{MR} = \frac{V^{2-n} d^n \rho}{K((3n + 1)/4n)^n 8^{n-1}} \quad (6)$$

Slatter and Lazarus (1993) proposed a Reynolds number for Herschel-Bulkley and Bingham model fluids stated as  $Re_2$ , comparable to that of Clapp Reynolds number as reported by Torrance (1963), but now incorporating the yield stress.  $Re_2$  is given by:

$$Re_2 = \frac{8\rho V^2}{\tau_y + k \left[\frac{8V}{d}\right]^n} \quad (7)$$

## 2. EXPERIMENTAL WORK

A rectangular steel-framed tank with clear Perspex walls (0.4 m, 0.4 m base and 0.6 m height) with a hole machined at the centre of a clear Perspex bottom was used to carry out all experiments. The tank was attached to the suspended weighbridge that was connected to the load cell. The load cell measured the change in volumetric mass over time. Two circular orifices of lengths 0 and 100 mm were each fitted in the center of the bottom of the tank flush with the inside surface. Both orifices were 20 mm in diameter; they had aspect ratios of 0, and 5 respectively. During experiments, the tank was manually filled with test fluids and thus a universal stopper was used to open and close the orifice when filling and emptying the tank. Newtonian fluids tested were water and various concentration of glycerine. Kaolin suspensions (yield pseudo-plastic/Herschel-Bulkley behaviour); CMC solutions (Power-Law behaviour) and bentonite suspensions (Bingham plastic behaviour) were prepared and tested at different concentrations in water. A Paar-Physica MCR300 rheometer was used to determine the rheological properties of the fluids. Coefficient of discharge ( $C_d$ ) values and appropriate non-Newtonian Reynolds numbers for each fluid and concentration were calculated. Table 1 shows the rheological properties of the test fluids.

Table 1

Fluid properties

Newtonian	Concentration %	Density (kg/m <sup>3</sup> )	$\mu$ (Pa.s)		
Water	-	1000	0.001		
Glycerine	100	1258	0.973		
	96	1248	0.304		
	93	1242	0.130		
	65	1179	0.019		
Non-Newtonian	Concentration %	Density (kg/m <sup>3</sup> )	Rheological properties		
			$\tau_y$ (Pa)	$K$ (Pa.s <sup>n</sup> )	$n$
CMC	2.4	1014		0.01	1
	5.2	1029	-	0.21	0.79
	6.6	1037	-	0.88	0.70
	7.6	1043	-	2.39	0.64
Kaolin	13.1	1217	8.90	0.07	0.72
	20.4	1336	39.42	3.96	0.36
Bentonite	3.8	1023	1.01	0.01	1
	7.2	1044	15.74	0.01	1
	7.3	1046	30.49	0.02	1

### 3. RESULTS AND DISCUSSIONS

#### 3.1 NEWTONIAN AND NON-NEWTONIAN FLUIDS

Figure 1 shows calibration results of  $L/d$  of 0 and 5 where  $C_d$  values are plotted against the  $Re$ . The results show that the  $C_d$  values obtained for  $L/d$  of 0 are smaller than those obtained for  $L/d$  of 5. The current study has an average  $C_d$  value of 0.60 and Dziubiński and Marcinkowski (2006) found an average  $C_d$  value of 0.62. These average  $C_d$  values are within  $\pm 1\%$  of the standard  $C_d$  value of 0.61 for circular sharp crested orifices (Yildirim, 2010). For an  $L/d$  of 5 the  $C_d$  values ranged from 0.77 to 0.78 with an average  $C_d$  of 0.78. There is a  $\pm 1\%$  difference compared to the average  $C_d$  value of 0.79 found by Çobanoğlu (2008).

Figure 2 displays the coefficient of the discharge as a function of the Reynolds number for both Newtonian and non-Newtonian fluids for an aspect ratio of 0. It is observed from Figure 2 that the trends for the  $C_d$  values in the laminar flow regime increased linearly with increase in the  $Re$  which occurred at  $20 < Re \leq 200$ ,  $70 < Re \leq 200$  and  $200 \leq Re \leq 700$  for flow trends of glycerine, CMC and kaolin respectively, thus forming three flow trends. The coefficients of discharge reached a peak of 0.68 in the transition zone, which in this case occurred at  $200 \leq Re \leq 1200$  for Newtonian fluids and of CMC and at  $700 \leq Re \leq 1200$  for kaolin. The turbulent flow region was in the range of  $2000 < Re \leq 70\,000$  for all the fluids, the  $C_d$  values became relatively constant within the range of 0.60 to 0.63 with an average  $C_d$  value of 0.61. These results are in agreement with results obtained by Dziubiński and Marcinkowski (2006) for Newtonian fluids where they obtained an average  $C_d$  value of 0.62 in the turbulent region. However, Dziubiński and Marcinkowski (2006) obtained an average  $C_d$  value of 0.67 for non-Newtonian turbulent flow data.

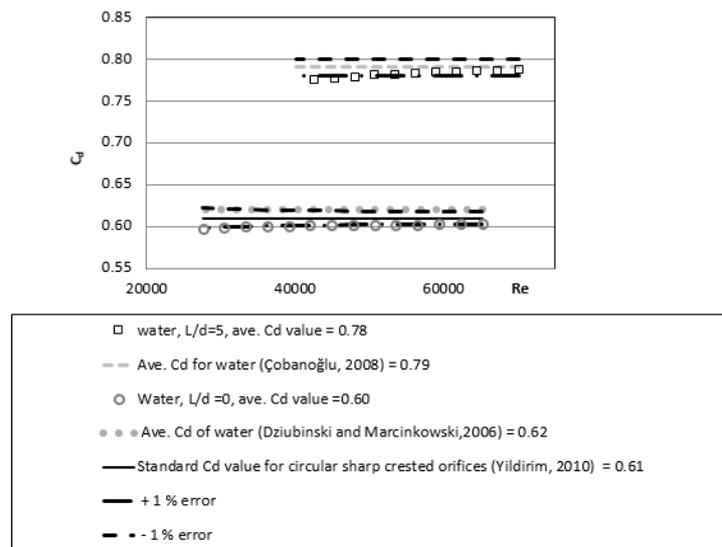


Figure 1. Orifice calibration for  $L/d = 0$  and  $L/d = 5$

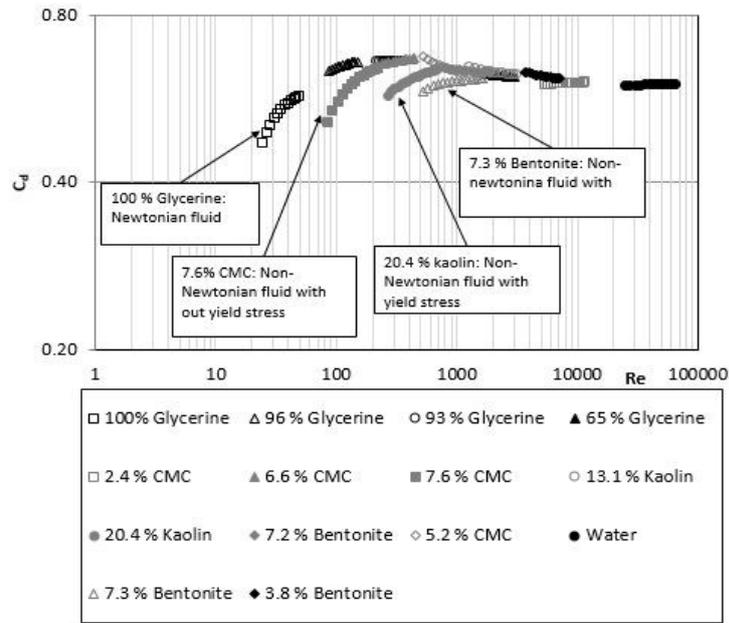


Figure 2. Coefficient of discharge versus Reynolds number for aspect ratio of 0

Figure 3 illustrates the relationship between the coefficient of discharge and the Reynolds number for both Newtonian and non-Newtonian fluids for an aspect ratio of 5. It is evident from Figure 3 that the trends for the  $C_d$  values in the laminar flow regime increased linearly with increase in the  $Re$  which occurred at  $40 < Re \leq 1000$  and  $100 < Re \leq 1000$  for Newtonian and non-Newtonian fluids respectively. The coefficients of discharge reached a peak of 0.74 in the transition zone, which in this case occurred at  $1000 \leq Re \leq 4000$  for both Newtonian and non-Newtonian fluids. For all the fluids, the  $C_d$  values became relatively constant at  $4000 < Re \leq 70000$  within the range of 0.75 to 0.78 with an average  $C_d$  value of 0.77. Dziubiński and Marcinkowski (2006) found an average  $C_d$  value of 0.62 and 0.67 for Newtonian and non-Newtonian fluids irrespective of the aspect ratio.

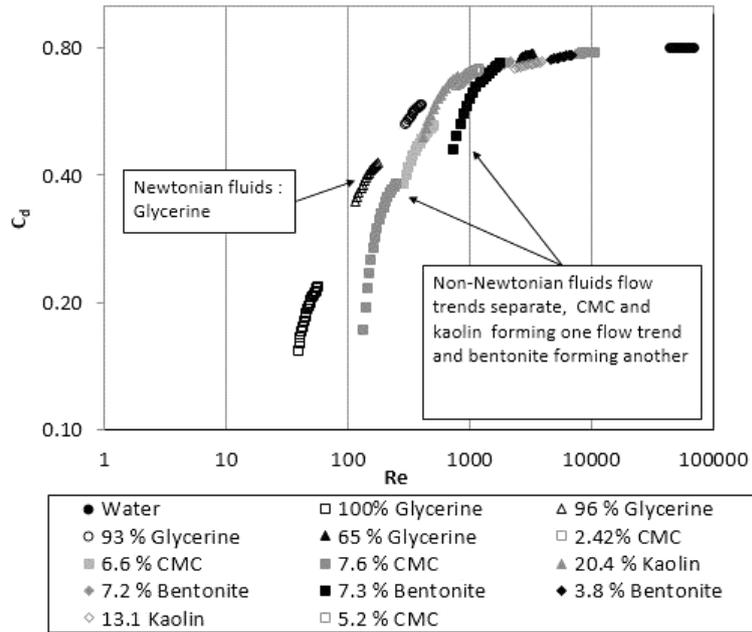


Figure 3. Coefficient of discharge versus Reynolds number for aspect ratio of 5.

Illustrated in Figure 4 (a) is Dziubinski and Marcinkowski (2006) data for aspect ratios 0, 1 and 3 and Figure 4 (b) is this study's experimental data for aspect ratios of 0 and 5. Dziubinski and Marcinkowski (2006) results, show that in the turbulent flows region, all aspect ratios have an average  $C_d$  value of 0.62 and 0.67 for Newtonian and non-Newtonian fluids respectively regardless of the varying aspect ratios. The current study indicates that in the turbulent flow, the  $C_d$  values are higher for  $L/d$  of 5 than for  $L/d$  of 0.

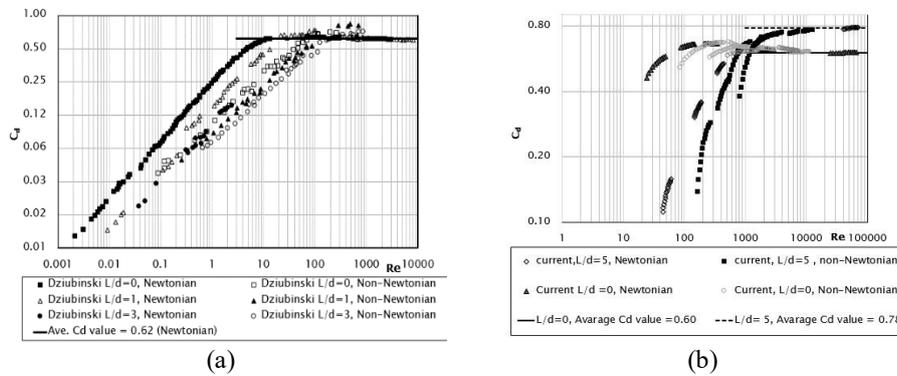


Figure 4. (a) Dziubinski and Marcinkowski (2006) data (b) This study's experimental data

#### 4. CONCLUSION

Calibration results showed that  $L/d$  ratio of 0 have a constant  $C_d$  value of 0.60 and it is within 1% error of standard average  $C_d$  value of sharp crested orifice of 0.61.  $L/d$  ratio of 5 has an average  $C_d$  value of 0.78 which is within  $\pm 1\%$  error compared to Çobanoğlu (2008) average  $C_d$  value. Therefore, the  $C_d$  value for water was found to increase with increasing  $L/d$  ratio. For an  $L/d$  of 0, in the laminar flow region, there are separate flow trends for glycerine solution, CMC solution, kaolin and bentonite suspensions. However, for  $L/d$  of 5 the three separate non-Newtonian fluids flow trends have collapsed into two flow trends and Newtonian fluids still have their own flow trend. This study extends the database on flow rate measurement of Newtonian and non-Newtonian fluids through orifices with various  $L/d$  ratios. Dziubinski and Marcinkowski (2006) used the highest  $L/d$  ratio of 3 and this study includes  $L/d$  ratio of 5. Dziubinski and Marcinkowski (2006) tested the Power-Law fluids and the current research includes Bingham and yield pseudoplastic fluids.

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